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Graph Cuts Based Phase Unwrapping

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Abstract—Phase unwrapping has always been a major concern in the field of image processing .In this paper,images having wrapped phase are unwrapped using the graph cut method . The case of eight neighbouring pixel pairs has been considered .Considering more neighbouring pixel pairs helps to get good unwrapped output because every step of unwrapping uses the adjoining pixels as its reference.The unwrapping of the wrapped image is done to make the data available for numerous applications.

Index Terms— Wrapped phase, interferometry, graph cuts, unwrapped phase, neighbouring pixels,interferometry,InSAR.

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1 INTRODUCTION

There are many applications that produce wrapped phase images. Examples of these are magnetic resonance imaging (MRI), synthetic aperture radar (SAR) and fringe pattern analysis. The wrapped phase images that are produced by these applications are not usable unless they are first unwrapped so as to form a continuous phase map. Phase unwrapping is a basic problem in signal processing. Different methods to achieve unlocked outputs from locked inputs have been discussed in [3] and [4]. They can be subdivided into several different categories, viz. network flow, branch-cut, minimum norm methods as explained in [5],[6],[7]. Phase unwrapping is the process of recovering the original image from the wrapped image. Medical, military and industrial applications require the extraction of the unwrapped phase image from the wrapped image. The phase which is obtained has phase jumps, which results in the formation of the wrapped phase. This wrapped phase is unusable until the phase discontinuities are removed. [1][2]Phase unwrapping is the reconstruction of a function on a grid by recovering the image from its wrapped state. We will refer to these two functions as the unwrapped and wrapped phases, respectively. Various problems such as discontinuities in phase, inadequate sampling rate, etc. affect the unwrapping process. In least-squares methods, unwrapping is achieved by minimizing the mean square deviation between the estimated and unknown discrete derivatives of the unwrapped phase. Least-squares methods are very efficient computationally when they make use of fast Fourier transform (FFT) techniques.

An interferometer is an instrument designed to produce optical interference fringes for measuring wavelengths, testing flat surfaces, measuring small distances, etc. Fig. 1 explains the principle of interferometry which makes use of the concept of superposition to combine or separate waves in a way that will cause the result of their combination to have some meaningful property that is diagnostic of the original state of the waves. This works because when two waves with the same frequency combine, the resulting pattern is determined by the phase difference between the two waves—waves that are in phase will undergo constructive interference while waves that are out of phase will undergo destructive interference.

Most interferometers use light or some other form of electromagnetic wave. Idealized interferometry involves determination of wavelength obtained by looking at interference fringes between two coherent beams recombined after travelling different distances. [3] Typically a single incoming beam of coherent light will be split into two identical beams by a grating or a partial mirror. Each of these beams will travel a different route, called a path, until they are recombined before arriving at a detector. The path difference in the distance travelled by each beam creates a phase difference between them. It is this introduced phase difference that creates the interference pattern between the initially identical waves. If a single beam has been split along two paths then the phase difference is diagnostic of anything that changes the phase along the paths. This could be a physical change in the path length itself or a change in the refractive index along the path [4][5]. In the past, astronomical interferometry used a single baseline to measure the amount of power on a particular small angular scale. Later astronomical interferometers which were used included arrays of telescopes. Interferometry techniques can be used to measure vibrational modes of solid bodies as well as the shape, flatness of optical surfaces, shifts in ground position or tilt which may indicate long term continental drift, shift in position of large suspended masses in the search for gravitational waves. Very long base-line interferometry can even be applied at cosmic distances.

Aerial photography is in use since several years to acquire data for the production of topographic maps. The efficiency of data collected is affected due to clouds and poor sunlight conditions. The problem is of particularly serious proportions in the cloud belt region within 15' of the equator and near the poles.

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The use of fine resolution radar systems has eliminated this problem, enabling efficient scheduling of flight operations, regardless poor weather conditions.

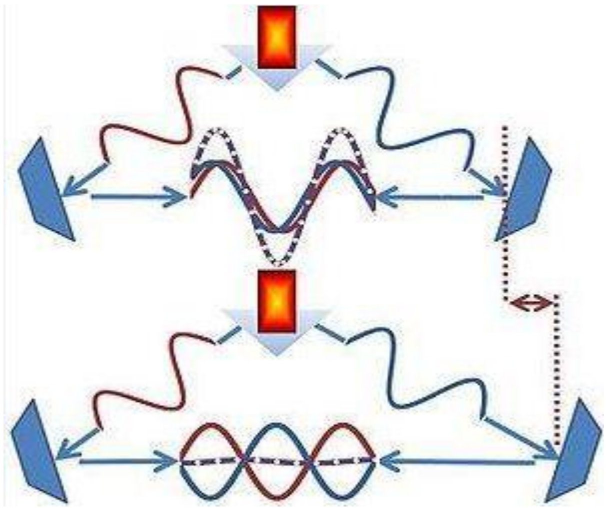

Fig. 1. Principle of interferometry[12]

SAR interferometric techniques fuse complex images captured by several antennas or at different instants to form interferograms . There are certain challenges in producing high resolution topographic images. This is overcome by a variety of imaging systems by using coherent radiation to illuminate objects. The scattered field returned back to the sensor carries information about the physical and geometrical properties of the imaged objects.

[6] Path following algorithms apply line integration schemes over the wrapped phase image, and basically rely on the assumption that Itoh condition holds along the integration path. Wherever this condition fails, different integration paths may lead to different unwrapped phase values. Path following methods directly apply the concept introduced in the Itoh algorithm, of discrete phase integration along a path. Given a starting pixel, for which the absolute phase is known, Itoh method prescribes on how to compute the absolute phase. The Bayesian approach relies on a data-observation mechanism model, as well as a prior knowledge of the phase to be modelled.

A wrapped phase image obtained from a simulated InSAR is shown in Fig.2. Images like in Fig.2 do not display much information when seen directly by human eyes. But, such images of geographical locations or physical structures convey a lot of information when processed.

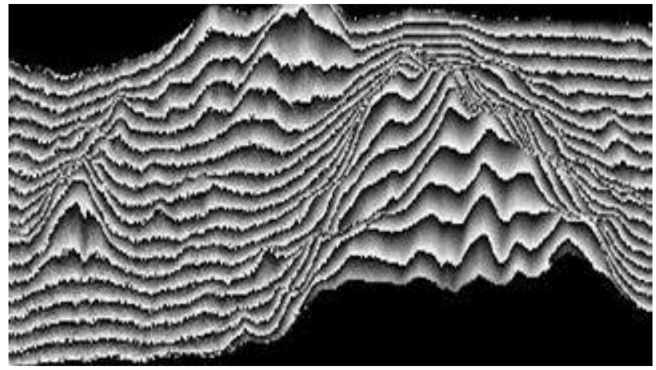

Fig. 2. Wrapped phase image obtained from a simulated InSAR [8]

2 PHASE UNWRAPPING BY THE USE OF GRAPH CUTS.

Fig.3 shows a graph cut method which shows a graph where there is a one-to-one correspondence between all the nodes. With such a graph, we can find a minimum cut of the graph. Fig.3 shows the structure of the graph in which first vertices are built and then the edges corresponding to each pair of adjoining pixels, and then those graphs are joined together .

Fig.4 shows the concept of graph cuts in which a cut is a partition of the vertices of a graph into two disjoint subsets. The cut-set of the cut is the set of edges whose end points are in different subsets of the partition. Edges are said to be crossing the cut if they are in its cut-set.

The size or weight of a cut is the number of edges crossing the cut. In a weighted graph , the same term is defined by the sum of the weights of the edges crossing the cut. In a flow network, an s-t cut is a cut that requires the source and the sink to be in different subsets, and its cut-set only consists of edges going from the source's side to the sink's side .[10]

The image with the wrapped phase is taken into consideration. Construct the initial graph and combine it with the main graph. First the initial elements of the map for every horizontal and vertical pairs of pixel are categorized depending on whether they belong to the starting bank or the destination bank . Both banks contain pixels in the form (i,j) . If a particular pair of pixels belongs to the starting bank (source), then we proceed further and finally stop when all the pixels belonging to that bank get exhausted. Similar procedure is followed at the other end. Based on this, the phase unwrapping is done using graph cut .[12]

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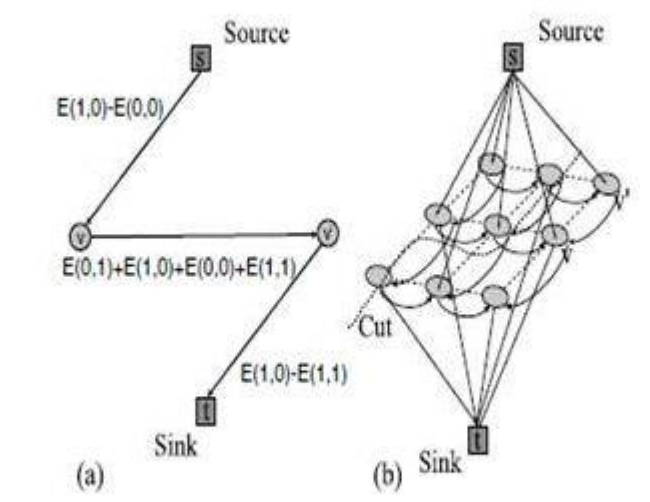

Fig. 3. (a) Elementary graph , where s and t represent source and sink, respectively, and v and v′ represent the two pixels involved in the energy term. In this case $E(1, 0)-E(0, 0) > 0$ and $E(1, 0)-E(1, 1) > 0$. (b) The graph obtained at the end results from adding elementary graphs [8]

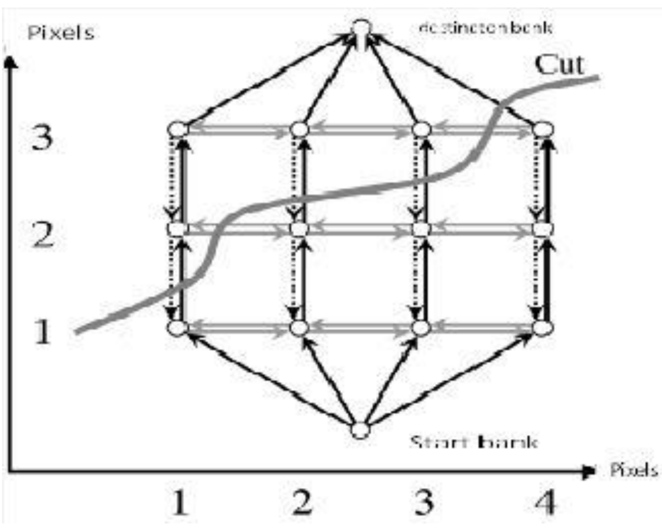

Fig. 4. The concept of graph cuts. [11]

In Fig.5, edge costs are reflected by their thickness. Consider a matrix consisting of pixels.Every pixel has its neighbours . Graph cut solutions for two or less neighbouring pixels have been obtained in the past. We have considered the case of eight neighbouring pixels.

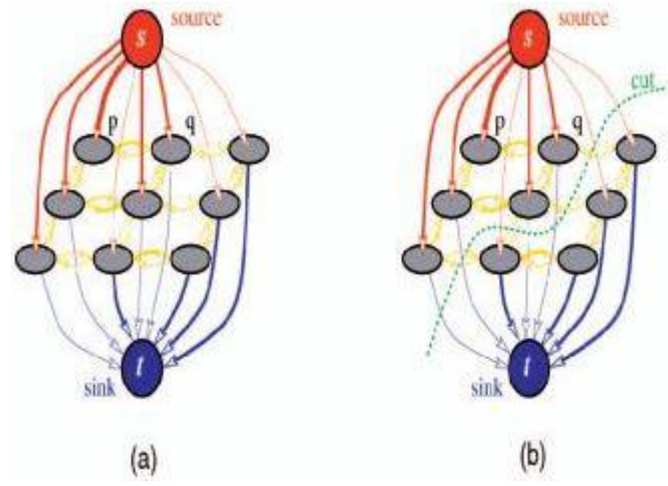

Fig. 5. (a) A directed graph. (b)A cut on the directed graph [9]

3 SIMULATION

All simulations have been done using MATLAB R2010a on a computer equipped with a 2.13 GHz Intel® Core™ i3 CPU. Fig.6 shows an image having phase wrap. Fig.7 shows an image with unwrapped phase using the graph cut method.

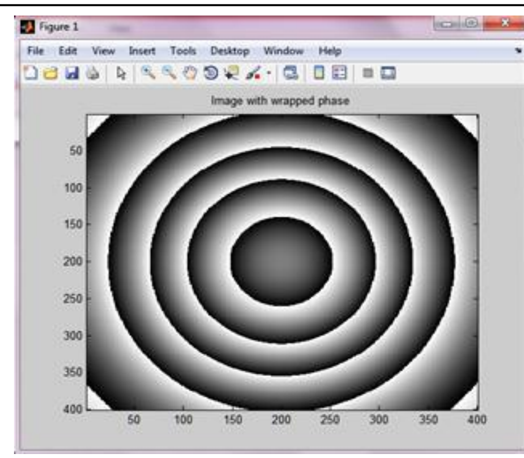

Fig. 6. Image having wrapped phase.

The output obtained in Fig.7 has been plotted as a three dimensional plot .The three dimensional plot of the image with the phase being unwrapped is shown in Fig.8.

Fig.9 shows a difference image which reveals the difference between the original image which had phase wraps and the final output which was recovered from the phase wraps.

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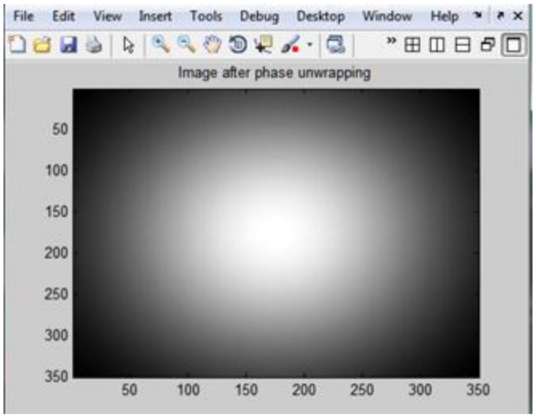

Fig. 7. Image with unwrapped phase

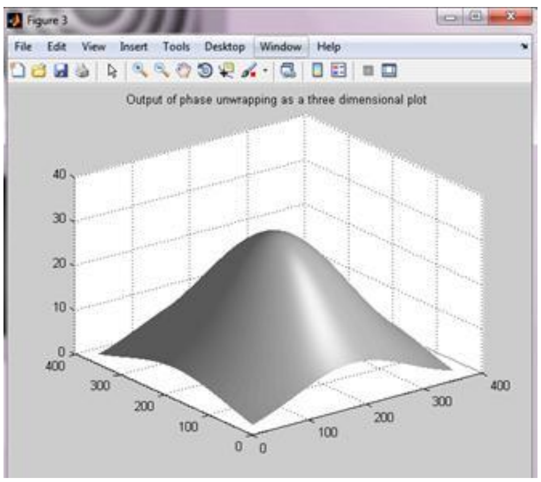

Fig. 8. Image which is phase unwrapped plotted as a three dimensional view.

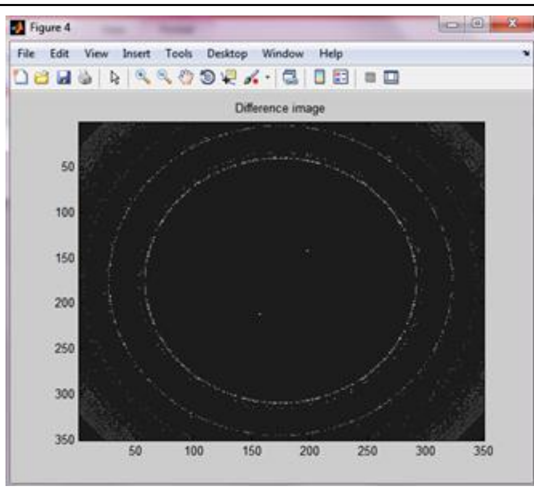

Fig. 9. Difference between the image having the wrapped phase & the unwrapped phase.

4 CONCLUSION

Phase unwrapping using graph cuts gives unwrapped phase image which can be useful for vaious applications such as medical,military,geo-studies and many more fields.The method of phase unwrapping using this technique gives a good result as the discontinuities in the phase can be overcome as smaller sections of the image are individually considered. The difference image clearly shows the eliminated error and explains that the original image with the unwanted phase wraps has been processed successfully to obtain the image without the phase wraps.

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